

Spontaneous jamming and unjamming in a hopper with multiple exit orifices

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We show that the flow of granular material inside a 2-dimensional flat bottomed hopper is altered significantly by having more than one exit orifice. For the hoppers with small orifice widths, intermittent flow through one orifice enables the resumption of flow through the adjacent jammed orifice, thus displaying a sequence of jamming and unjamming events. Using discrete element simulations, we show that the total amount of granular material (i.e. avalanche size) emanating from all the orifices combined can be enhanced by about an order of magnitude difference by simply adjusting the inter-orifice distance. The unjamming is driven primarily by fluctuations alone when the inter-orifice distance is large, but when the orifices are brought close enough, the fluctuations along with the mean flow cause the flow unjamming.

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An assembly of discrete, non-cohesive particles, aka dry granular media, while trying to flow through a narrow opening can clog or jam, an occurrence widely observed in particle drainage through silos and hoppers which are used ubiquitously in several industrial applications. The jamming at the orifice is caused due to few particles forming a stable arch at the exit [1, 2] eventually causing the entire system to halt abruptly. The particles constituting the arch can be from anywhere in the system [2] and while occurrence of the arch is quite unpredictable, some information can be obtained through the spatial distribution of the velocity fluctuations in the system [3]. The shape of the arch, though, can be predicted quite well using a random walk model [1].

For a three dimensional hopper there exists a critical orifice width above which the system never jams [4] while no such limit exists for a two dimensional system which can jam for large enough orifice widths [5]. It has been shown that the output flow rate from a hopper can be increased as large as 10% by placing suitable inserts within at appropriate locations [6–8]. This also decreases the probability of jamming by about two orders of magnitude [9]. These studies have shown that the reduction in the tendency to jam is due to the reduction of the pressure in the region of arch formation. The motivation for placing inserts is derived from parallel interesting studies carried out to alter the flow behavior of pedestrians from a crowded room [10, 11].

Over here we report an interesting observation about the jamming behavior in a 2-dimensional hopper having two exit orifices of same width placed far away from each other. Whenever one of the orifice jams, which is expected given the orifice width incorporated, it unjams spontaneously if the flow is occurring through other orifice. Effectively, flow continues to occur through both orifices, alternately or together, for much longer duration than expected for a hopper with single orifice. The overall duration of flow, consequently the tendency of jamming, can be altered simply by changing the inter-

orifice distance. Such non-local interaction between regions exhibiting differing flow behavior has been observed previously, but in different configurations [12, 13] and under different flow conditions. In these studies it was shown that the origin of such non-local interaction can be attributed to a self activated process within which the stress fluctuations induced by a localized shear causes the material to yield and flow elsewhere [13].

In the present system, we conjecture that the observed behavior of spontaneous unjamming occurs due to rearrangement of the particles in the jammed region above one orifice. This unjamming behavior can be correlated with the fluctuations induced in the system due to flow from the nearby orifice. We believe that such non-local interaction between two widely separate regions can be used to systematically alter the jamming behavior in a hopper non-intrusively which could be quite significant for several industrial operations. We have explored this behavior in great detail through DEM simulations of soft particles. We measure the mean avalanche size for varying conditions and show their correlation with the fluctuations measured as root mean squared (r.m.s.) values.

The DEM simulations are carried out using the Large Atomic/Molecular Massively Parallel Simulator

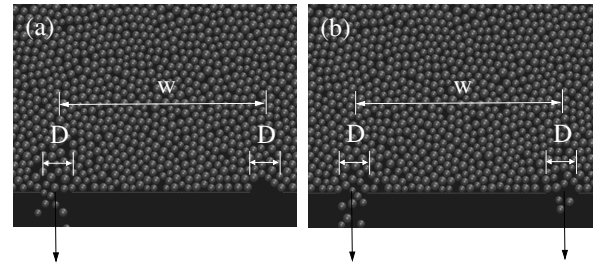


FIG. 1. Sample snapshots of the flow occurrence in a two-orifice hopper for a specified w and D . (a) Flow occurring through the left orifice while the right orifice is jammed. (b) Spontaneous flow re-initialization through the right orifice at a later time.

(LAMMPS) developed at Sandia National Laboratories [14, 15]. The simulation employs Hookean force between two contacting particles described in detail elsewhere [16]. All the simulation parameters are the same as used in the systematic study of hopper flows carried out previously [16] except for a higher normal elastic constant ($k_n = 2 \times 10^6 mgd$) which corresponds to a more stiffer particle. The inter-particle friction coefficient (μ) is varied from 0.2 to 0.8 with no qualitative difference between the results. Over here, we report the results obtained for $\mu = 0.5$.

The simulation geometry consists of a 2-dimensional hopper of height about 1.2 times its width and thickness $1d$, where d is the mean particle diameter with a poly-dispersity of 15 %. The sidewalls are created out of the largest particles (rough wall) while the bottom surface is created from the smallest particles (relatively smooth wall) by freezing the particles so that their translational and angular velocities are kept zero throughout the simulation. The hopper has two orifices of width D placed at an inter-orifice distance w , both defined in terms of mean particle diameter d . The hopper width is large enough to prevent any confinement effects due to their proximity to either orifice. The fill height is maintained constant by re-positioning the particles exiting from the orifice just above the free surface. The hopper is filled using the sedimentation method as suggested previously [17] in which a dilute packing of non-overlapping particles is created in a simulation box and allowed to settle under the influence of gravity. The simulation is run for a significant time so that the kinetic energy per particle is less than $10^{-8} mgd$ resulting into a quiescent packing of desired fill height H in the hopper.

The flow through the hopper is initiated by opening both the orifices simultaneously. The orifice width is chosen to be small enough to cause jamming after certain period of flow. After the flow is initialized, either of the orifice gets jammed but unjams again spontaneously. Note that this unjamming would not have been possible in the absence of second orifice through which the flow occurs for a very short duration before jamming itself. The jamming-unjamming sequence can, thus, flip from one orifice to other. Effectively the flow occurs through either one or both the orifices at any given time. After few of these jamming-unjamming sequences, the flowing orifice jams before it can unjam the other orifice and the overall flow stops. It is quite evident that the particles above the flowing orifice transmit some information to the jammed orifice causing it to unjam again. However, this information is available only for a short duration before the flowing orifice jams on its own. After both orifices jam, the flow is re-initiated by removing 2 – 3 particles from each of the arch. This procedure is continued to get significant number of jamming events. The total number of particles flowing out from the hopper from the instant both orifices are opened until both are

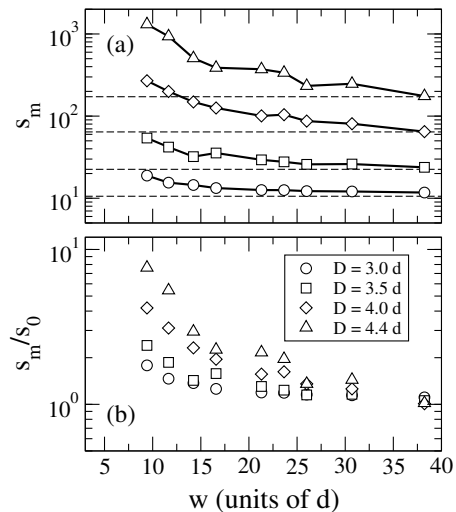


FIG. 2. Jamming characteristics for varying distances (w) between orifices of different widths (D). (a) Mean avalanche size per unit orifice (s_m) plotted against the inter-orifice distance (w). Dashed lines corresponds to the avalanche size (s_0) for a hopper having single orifice of width D . (b) Scaled mean avalanche size (s_m/s_0) plotted against the inter-orifice distance (w).

jammed is defined as an avalanche size (s). The value of s is found to depend on the values of D and w which we discuss next. A sample snapshot of particles flowing through one orifice while other being jammed is shown in Fig. 1a. A small time later, the jammed orifice starts to flow spontaneously as shown in Fig. 1b.

The distribution of avalanche sizes (s) for a given D and w , normalized by the mean avalanche size $\langle s \rangle$, exhibits an exponential tail for all cases which is typical of the random nature of the discrete avalanche events [9]. Fig. 2a shows the mean avalanche size per orifice ($s_m = \langle s \rangle / 2$) obtained for four different values of D and several inter-orifice distance (w). The behavior is qualitatively similar for all the orifice sizes. The avalanche size (s_m) decreases monotonically with increasing w and at infinitely large w it asymptotically approaches a constant value, which corresponds to that for a single orifice hopper (see Fig. 3b). In this scenario, the two orifices will function unaware of existence of the other and no information is exchanged between the respective flowing regions. For w larger than those shown in Fig. 2a (but not studied), an unjamming event can happen, but with much lesser probability. Now with decreasing w , the value of s_m increases and it grows quite rapidly for small enough w . This is the consequence of the flow from one orifice aiding that through the other in some way, thus, effectively increasing the total time period over which flow occurs, hence larger s_m . In the limit the two orifices are very close to each other (w of the order of D or lower), the flow now occurs as if through a wider orifice

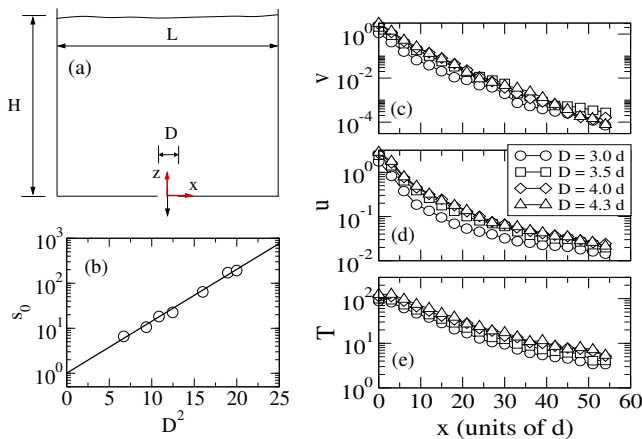


FIG. 3. (a) Schematic of a single orifice hopper of width L and height H . (b) Variation of mean avalanche size (s_0) for a hopper with single orifice of width D . Variation of (c) mean velocity v , (d) r.m.s. velocity u and (e) r.m.s. normal stress T along the horizontal direction (x) obtained over a region $3d$ in z -direction for four different orifice widths. Please refer to text for the method to obtain mean and rms quantities. Mean velocity and rms velocity are measured in units of d/τ , while rms stress is measured in units of mg/d^2 .

($\sim 2D$). Eventually it approaches the asymptotic limit for a single orifice of width $2D$. Within the entire range of w , the value of s_m is observed to vary by almost an order of magnitude difference for the largest orifice width considered. The mean avalanche size (s_m) normalized by the single orifice avalanche size (s_0) is shown in Fig. 2b. The data shows reasonable collapse for $w > 20d$, while increasing scatter is observed with decreasing w , which, perhaps, indicates a non-linear dependence on the orifice width (D) and interacting flow fields which cannot be scaled out by simple normalizing.

To elucidate the origin of the enhanced avalanche sizes, we perform simulations in a hopper fitted with single orifice for different D (see Fig. 3a). The width of the hopper is more than twice the maximum distance (w) used in two orifice system. For every orifice size, several avalanches are obtained by re-initiating the flow post jamming. The mean avalanche size shows an exponential-squared dependence on the orifice size ($e^{[0.26D^2]}$) as shown in Fig. 3b and is in accordance with experimental results obtained previously for a 2-dimensional system [5] suggesting of the absence of a critical orifice width (D) separating the flowing and jamming conditions.

We next obtain the profiles of mean velocity, velocity fluctuations and normal stress fluctuations of the particles along the x -direction. To calculate these quantities, we save snapshots of particle positions at intervals of 0.0025τ , where $\tau = \sqrt{d/g}$, g is the gravitational acceleration and integration time step used in the simulations is $\delta t = 2.5 \times 10^{-5}\tau$. The mean velocity is defined as $v(x) = \sqrt{\langle c_x \rangle^2 + \langle c_z \rangle^2}$, while the average velocity fluctuations

are measured in terms of root mean squared (rms) velocity defined as $u(x) = \sqrt{[\langle c_x^2 \rangle - \langle c_x \rangle^2] + [\langle c_z^2 \rangle - \langle c_z \rangle^2]}$. Here, c_x and c_z are the instantaneous horizontal and vertical velocity components of every particle obtained from the displacements between two successive snapshots. The simulation algorithm also outputs the horizontal (σ_x) and vertical (σ_z) component of computed normal stress on each particle within each snapshot. The normal stress fluctuations are captured in terms of r.m.s. stress defined as $T(x) = \sqrt{[\langle \sigma_x^2 \rangle - \langle \sigma_x \rangle^2] + [\langle \sigma_z^2 \rangle - \langle \sigma_z \rangle^2]}$. In all the above calculations, $\langle \rangle$ represents a temporal average over several time instants of flow and spatial average over a $3d$ region in z -direction located about $4d$ above the orifice. This spatial region is chosen so as to capture the essential dynamics in a simple manner. All the three quantities are shown in Fig. 3c,d,e for the four orifice widths. The profiles are symmetric about the orifice and hence only the right half of the profiles are shown in each case.

The magnitude of all the quantities increase with an increase in the orifice width which is expected given the faster flow and consequently, more collisions between particles. The profiles show similar behavior for different orifice sizes and collapse quite nicely (not shown) when normalized by the respective quantity at the $x = 0$. The mean velocity decays much rapidly with distance from the orifice while the r.m.s. velocity and stress show a much gradual decay. The ratio of r.m.s. to the mean velocity increases progressively away from the orifice and reaches a value of about 100 by $x = 45d$. Almost similar numbers are obtained for all the orifice sizes studied.

Now consider the scenario of a two orifice hopper with one of the orifice located at $x = 0$ which is in unjammed (flowing) state. Provided the other orifice located at some distance (w or x) is in a jammed state, the flow profile due to this orifice will be the same as shown in Fig. 3c-e. The occurrence of flow through the unjammed orifice over a time period (which corresponds to the time interval between jamming and unjamming instances of second orifice), causes particle rearrangements in the system and breaks the arch above second orifice leading to flow. For an inter-orifice distance lesser than $10d$, with the mean and r.m.s. velocity magnitudes being of same order it is difficult to clearly isolate the effect of each on the spontaneous unjamming behavior. However, for larger inter-orifice distances ($> 20d$), the mean velocity is over an order of magnitude lower than the rms velocity and the fluctuations are expected to dominate the spontaneous unjamming behavior. It is expected that the fluctuations too will decay at infinitely large distances and will not be able to re-initiate flow in the jammed orifice. The flow and jamming behavior of the two orifices in that case resemble that from the two isolated orifices unaware of the other's existence. The mean avalanche size then approaches that for a single orifice (dashed lines in Fig. 2b).

Given the observed dependence of the mean avalanche

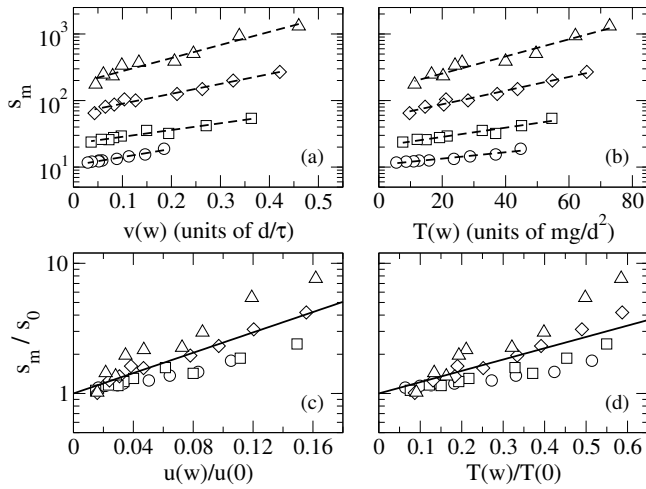


FIG. 4. Correlation between mean avalanche size per unit orifice width (s_m) and (a) the r.m.s. velocity and (b) the r.m.s normal stress. All quantities obtained at distance (w). Dashed lines show an exponential fit. (c, d) Data in (a) and (b) respectively after normalizing s_m with mean avalanche size (s_0) for a hopper with single orifice. The solid line in (c) and (d) shows an exponential fit.

size (see Fig. 2a) and that of the flow variables (see Fig. 3c,d,e) on the inter-orifice distance, we now proceed to determine the relation between these quantities. We consider data only for inter-orifice distance greater than $10d$ beyond which the mean flow decreases rapidly compared to fluctuations. Fig. 4a and b shows the avalanche sizes (s_m) for different inter-orifice distances (w) plotted against the rms velocity $u(w)$ and rms normal stress $T(w)$, respectively. All quantities are evaluated at the same distance (w or x) in all the cases. An exponential relation (shown as dashed lines) is observed for all the cases where with the y-intercept approximately equal to the avalanche size (s_0) for a single orifice hopper (i.e. two orifices infinitely far away from each other). The normalized mean avalanche size (s_m/s_0) plotted against the normalized r.m.s velocity and normalized r.m.s. normal stress values is shown in Fig. 4c and d, respectively. The rms velocity and normal stress are normalized with their corresponding values at $x = 0$. The collapse is much better than that observed in Fig. 2b which indicates that the fluctuations do play a role and w is not the only parameter influencing the flow behavior. The solid line shows an exponential fit. For smaller $u(w)/u(0)$ and $T(w)/T(0)$ ($w > 20d$), where the primary quantity responsible for unjamming is fluctuations, the data collapse is quite good. However, the scaling shows increasing scatter at increasing values of $u(w)/u(0)$ and $T(w)/T(0)$ (or smaller w) which perhaps is indicative of the more complex dependence on the mean flow in addition to fluctuations and the inter-orifice width. Similar scaling behavior for both, r.m.s. velocity and r.m.s. normal stress

data, is not quite surprising as both represent average fluctuations in the system and are quite inter-related to each other. Fluctuation of velocity is expected to generate collisions between particles leading to fluctuations in stresses and vice-versa.

In conclusion, our study suggest that the jamming occurrences within a hopper can be altered non-intrusively using multiple orifices by varying the inter-orifice distance. The hopper can be made to flow for prolonged duration for a small enough orifice size by having another orifice of same size at different distances. We observe that the fluctuations arising locally cause rearrangements of particles in the region located far away which leads to spontaneous unjamming. We would also like to note that while the results have been reported for a strictly 2-dimensional system, similar qualitative behavior is observed in simulations of 3-dimensional or quasi-3d systems. This mechanism of spontaneous jamming-unjamming can be of immense importance for (a) more detailed exploration of jamming characteristics for granular systems studied previously [18–21] and (b) investigating other disordered systems which exhibit jamming, viz. bubbles escaping from an orifice [22] or colloidal hard spheres flowing through a constriction [23]. As a system of practical utility, the multi-orifice hopper can be operated as an efficient mixing device [24] wherein the mixing can be initialized through interaction between different zones of an hopper through either random or controlled closing and opening of orifice.

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